The Mathematical Association of Victoria

Further Mathematics

2006 Written Examinations

Solutions

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2006 Further Mathematics
Written Examination 1 (Facts, skills and applications)
Suggested answers and solutions

SECTION A (Multiple-Choice**)**
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Core
1. **D** 2. **B** 3. **E** 4. **C** 5. **D**
6. **B** 7. **A** 8. **C** 9. **A** 10. **C**
11. **D** 12. **C** 13. **A**

SECTION B (Multiple-Choice**)**
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Module 2 Geometry and trigonometry
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Module 4 Business-related `mathematics
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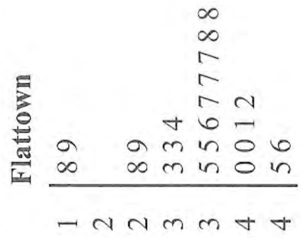
Module 6 Matrices
1. **B** 2. **D** 3. **A** 4. **B** 5. **D**
6. **E** 7. **C** 8. **E** 9. **B**

Core
Question 1 [D]
Temperature (° Celsius) and Town (Beachside and Flattown) are numerical and categorical variables respectively.

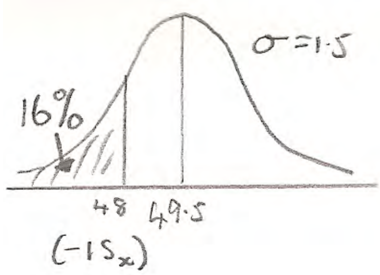
Question 2 [B]
Range = Maximum – Minimum
$= 38 - 15$
$= 23$

Question 3 [E]



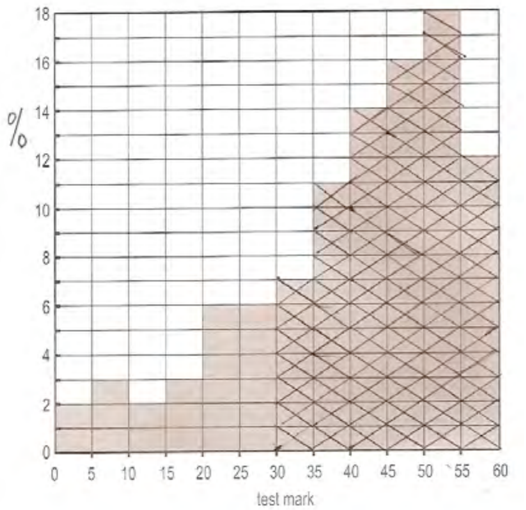
The distribution of maximum temperatures for Flattown is best described as approximately symmetric with outliers.

Question 4 [C]



The shaded area less than 48 cm is approximately 16%.
The number of boys with head circumference of less than 48.0 cm is closest to $16\% \times 400 = 64$

Question 5 [D]



The % of students who passed the test (ie 30 or above) is

$7 + 11 + 14 + 16 + 18 + 12 = 78\%$

Question 6 [B]

The median point lies at 50%.

The % of students from 0 to 30-35 is

$2 + 3 + 2 + 3 + 6 + 6 + 7 + 11 = 40$

The % of students from 0 to 40 - 45 is

$2 + 3 + 2 + 3 + 6 + 6 + 7 + 11 + 14 = 54$

So the median lies between 40-45.

Question 7 [A]

$r = -0.5675,\ \bar{x} = 4.56,\ s_x = 2.61,$

$\bar{y} = 23.93$ and $s_y = 6.98$

$$b = r\frac{S_y}{S_x} = -0.5675\left(\frac{6.98}{2.61}\right) \approx -1.52$$

$$a = \bar{y} - b\bar{x} = 23.93 + 1.52(4.56) \approx 30.9$$

$$y = 30.9 - 1.52x$$

Question 8 [C]

Actual

When Waist = 80 cm, weight = 67 kg

Estimate

Weight = $-20 + (1.11 \times 80) = 68.8$

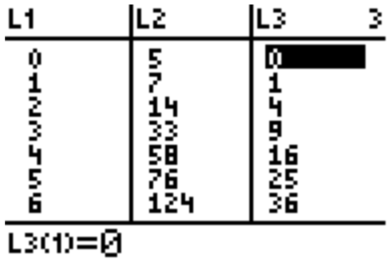
Residual =

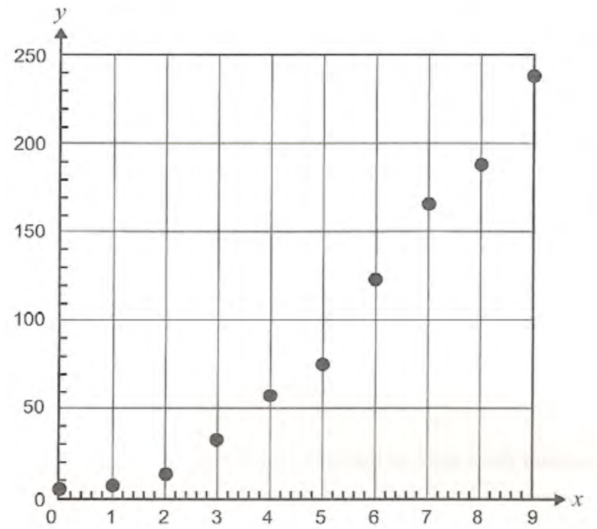
$y_{act} - y_{est} = 67 - 68.8 = -1.8 \approx -2$ kg

Question 9 [A]

Input data into 2 lists.

x into L_1 and y into L_2. $L_3 = x^2$



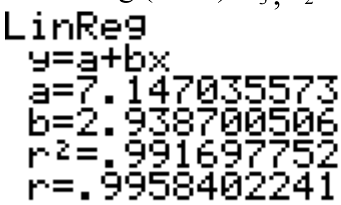


To find the least squares regression line for the transformed data:

STAT

CALC

8: LinReg (a+bx) L_3, L_2



So the equation is $y = 7.1 + 2.9x^2$

Question 10 [C]

Mar	April	May
35	99	75

Three mean moving average for April

$$= \frac{35 + 99 + 75}{3} = 69.66... \approx 70$$

Question 11 **[D]**
The seasonal indices add to 12.
The seasonal index for October
$= 12 - (1.30 + 1.21 + 1.00 + 0.95 + +0.95$
$+ 0.86 + 0.86 + 0.89 + 0.94 + 0.99 + 1.07))$
$= 0.98$

Question 12 **[C]**
$$\text{Deseasonalised} = \frac{\text{Actual}}{\text{Seasonal Index}}$$
$$= \frac{330}{0.94} = 351$$

Question 13 **[A]**
Deseasonalised :
$= 373.3 - (3.38 \times 6)$
$= 353.0$
Actual is Deseasonalised $\times$ Seasonal Index
$= 353.02 \times 0.86$
≈ 304

Module 1 Number patterns

Question 1 **[D]**
An arithmetic sequence has a common difference d.
For $-4, -1, 2, 5, 8, \ldots\ldots,$
$d = -1 - (-4) = 3$

Question 2 **[C]**
The first 3 terms of a geometric sequence are $6,\ x,\ 54$.
$\frac{x}{6} = \frac{54}{x}$
$x^2 = 324$
$x = 18$

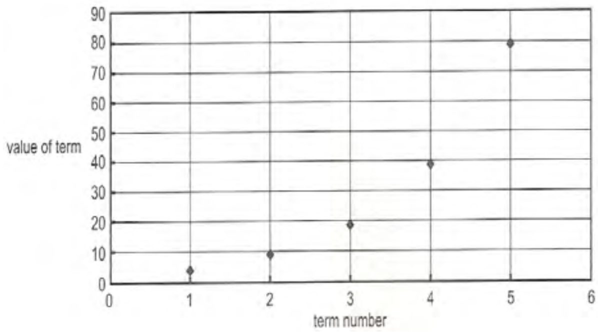
Question 3 **[C]**
At the start of the second year, the farmer has $(50 \times 1.84) - 40 = 52$ sheep

Question 4 **[A]**
$S_1 = 50,\ d = -40,\ r = 1.84$
The difference equation which models this growth of sheep over time is:
$S_{n+1} = 1.84 S_n - 40$ where $S_1 = 50$

Question 5 **[D]**
For $f_{n+1} - f_n = 5$ where $f_1 = -1$
$f_2 = 5 + f_1 = 5 - 1 = 4$
$f_3 = 5 + f_2 = 5 + 4 = 9$
So the sequence is $-1, 4, 9, \ldots\ldots$

Question 6 **[E]**
$a = 12,\quad r = 1.03$
$t_{15} = 12 \times 1.03^{14} = 18.15 \approx 18.2$

Question 7 **[B]**



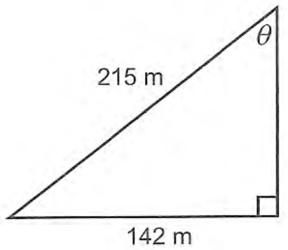
As the graph is non linear, the difference equation is **not** arithmetic (A, C and D)
The sequence is $4, 9, 19, 39, 79$
So the difference equation is
$t_{n+1} = 2t_n + 1,\quad t_1 = 4$

Question 8 **[D]**
$t_n = t_{n-1} + t_{n-2}$ where $t_1 = 1$ and $t_2 = 2$
$t_3 = t_2 + t_1 = 2 + 1 = 3$
$t_4 = t_3 + t_2 = 3 + 2 = 5$
$t_5 = t_4 + t_3 = 5 + 3 = 8$
So the total number of stamps after 5 weeks $= 1 + 2 + 3 + 5 + 8 = 19$

Question 9 **[E]**
$a = 73.4,\ r = \frac{380}{400} = \frac{361}{380} = 0.95$
$S_\infty = \frac{400}{1 - 0.95} = 8000g = 8$ kg
Roh's eventual body weight will be $73.4 + 8 = 81.4$ kg

Module 2 Geometry and trigonometry

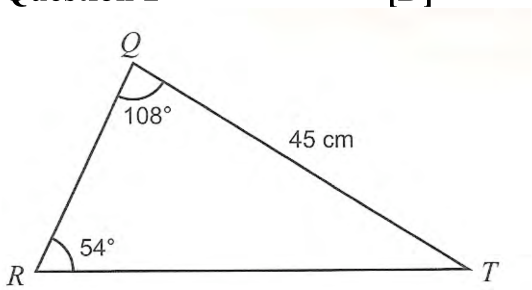
Question 1 [B]



$$\sin\theta = \frac{142}{215}$$

$$\theta = \sin^{-1}\frac{142}{215} \approx 41°$$

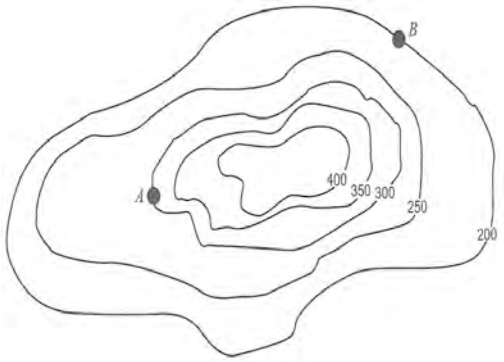
Question 2 [D]



$$\frac{45}{\sin 54^0} = \frac{RT}{\sin 108^0}$$

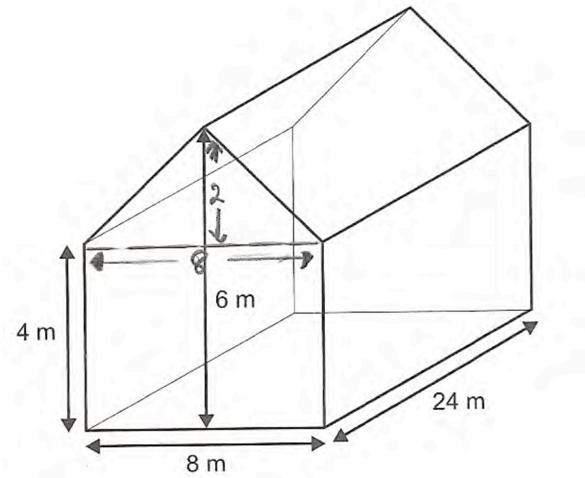
$$RT = \frac{45\sin 108^0}{\sin 54^0} = 52.9° \approx 53°$$

Question 3 [A]



The difference in height between A and B is $300 - 200 = 100\,\text{m}$

Question 4 [B]



$$V = (8\times4\times24) + (0.5\times2\times8\times24) = 960\,\text{m}^3$$

Question 5 [E]

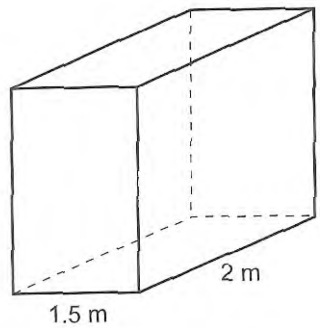
Using Heron's Formula:

$$S = \frac{1}{2}(a+b+c) = \frac{1}{2}(36+58+42) = 68$$

$$A = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{68(68-36)(68-58)(68-42)}$$

Question 6 [D]



$$V = l\times w\times h$$

$$6 = 1.5\times2\times h,$$

So $h = 2$

$$TSA = 2(1.5\times2) + 2(2\times2) + 2(1.5\times2)$$

$$= 6 + 8 + 6$$

$$= 20\,\text{m}^2$$

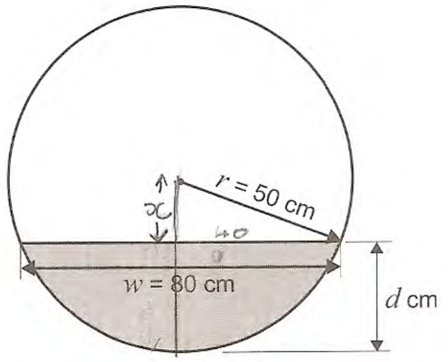
Question 7 [E]

Using Similar Triangles ΔCAB and ΔEDB

$$\frac{24}{36} = \frac{DE}{27}$$

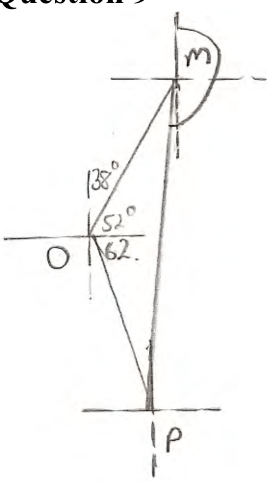
$$DE = \frac{24 \times 27}{36} = 18\text{ cm}$$

Question 8 [A]



$$x^2 = 50^2 - 40^2$$
$$= 2500 - 1600$$
$$= 900$$
$$x = 30$$
$$d = 50 - 30 = 20\text{ cm}$$

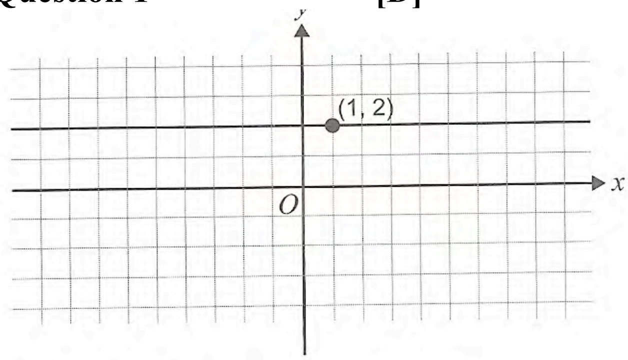
Question 9 [D]



From the diagram, the bearing of P from M is between 180° and 270°

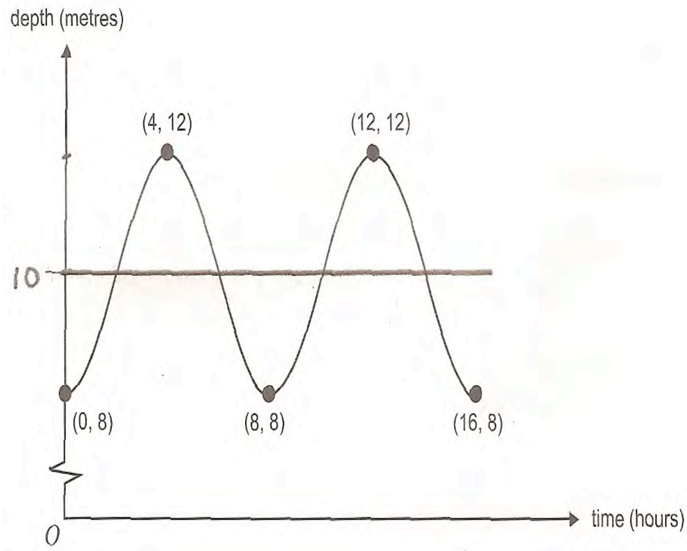
Module 3 Graphs and relations

Question 1 [D]



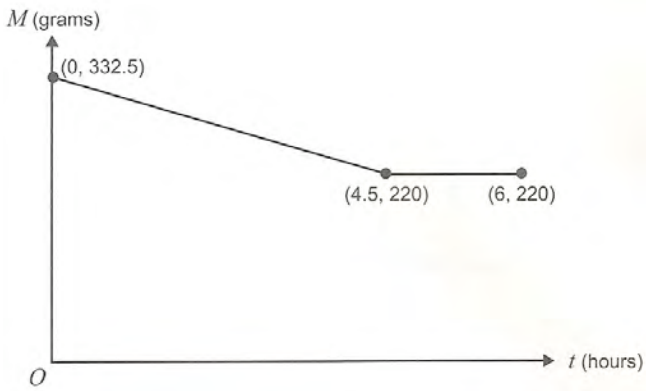
The equation of the line passing through the point (1,2) is $y = 2$

Question 2 [E]



From the graph, there are 4 times when the depth of the water is 10m.

Question 3 [C]



From the graph it can be seen that the lamp runs out of gas after 4.5 hours.

Question 4 **[A]**

The gradient for the straight line joining (0, 332.5) to (4.5, 220) is:

$$\frac{220-332.5}{4.5}=-25$$

The M -intercept is 332.5

The gradient from (4.5, 220) onwards is 0.

The rule would be

$$M=\begin{cases}332.5-25t & \text{for} \quad 0\le t\le 4.5\\ 220 & \text{for} \quad 4.5<t\le 6\end{cases}$$

Question 5 **[B]**

The equation $12x-4y=0$ can be rewritten as $y=3x$.

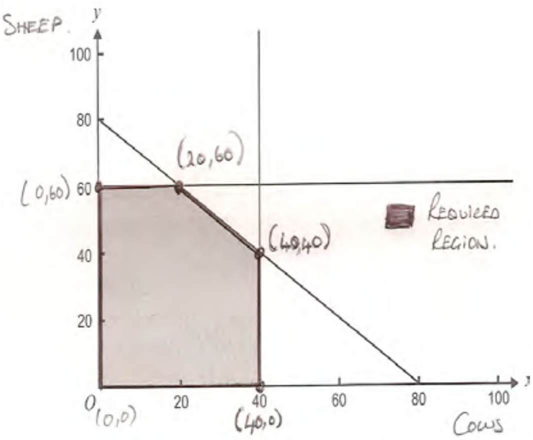
So the line **does not** have a slope of 12.

Question 6 **[C]**

The point of intersection of two lines is (2, -2) so (2, -2) must be a point on both lines.

Substituting (2, -2) into $2x+2y=0$ gives $4-4=0$ which is true. So one of these two lines could be $2x+2y=0$.

Question 7 **[E]**



One of the constraints defining the feasible region indicates that the total number of cows and sheep cannot exceed 80.

Question 8 **[A]**

The cost equation is $C=400+50x$.

Breakeven occurs when Cost = Sell

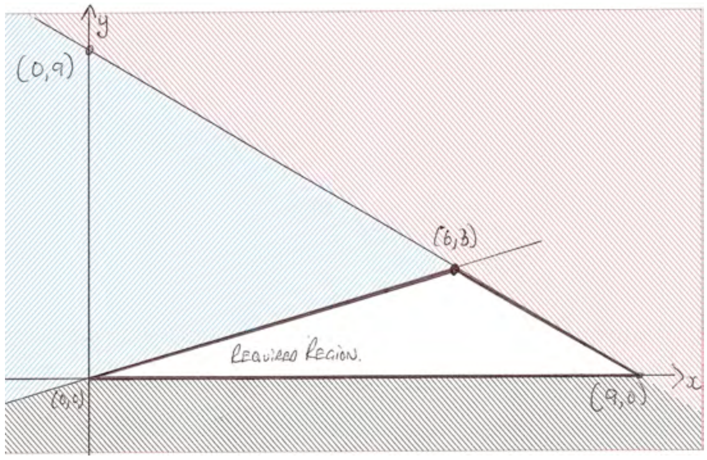
For 10 frying pans the cost and selling prices are:

$Cost=400+50\times 10=900$

$Sell=90\times 10=900$

So the manufacturer could break even by selling 10 frying pans at $90 each.

Question 9 **[C]**



(6,2) lies within the feasible region.

Module 4 Business-related mathematics

Question 1 **[B]**

$$I=\frac{\Pr t}{100}=\frac{4000\times 5\times 1}{100}=200$$

$200 interest is earned in the first year.

Question 2 **[C]**

Minimum balance for October is $473.92.

$$I=\frac{\Pr t}{100}=\frac{473.92\times 0.15\times 1}{100\times 1}=\$0.71$$

Interest paid for October is $0.71.

Question 3 **[E]**

A perpetuity is an annuity where a permanently invested sum of money provides regular payments that continue forever.

Yearly interest $= \$584 \times 12 = \$7008$

$$P = \frac{100I}{rt} = \frac{100 \times 7008}{6.2 \times 1} \approx \$113000$$

Grandpa needs to invest \$113000 in the perpetuity.

Question 4 **[B]**

$1.1x = 825$

$$x = \frac{825}{1.1} = 750 \quad \text{(without GST)}$$

GST = 825-750 = \$75.00

Question 5 **[E]**

Total Number of copies

$$= \frac{48000 - 21000}{0.04}$$

$= 675000$

Question 6 **[C]**

Principal = \$2000 −\$200(deposit)
= \$1800

Total Instalments = $36 \times 68 = \$2448$

$I = 2448 - 1800 = 648$

$$r_f = \frac{100 \times 648}{1800 \times 3} = 12\%$$

Annual flat rate of interest is 12%.

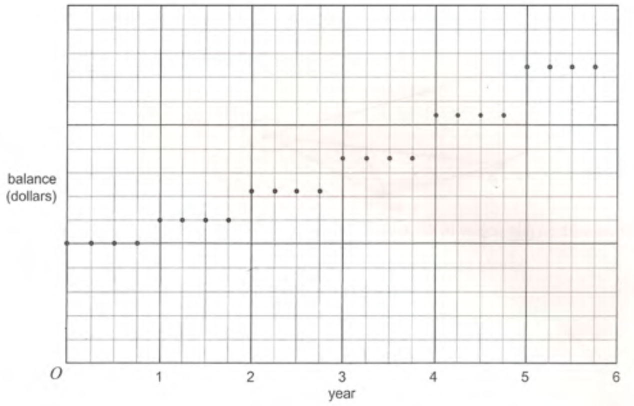
Question 7 **[D]**

Price paid for lawn mower = \$368

\$368 + \$80 (Trade in) = \$448

$$\frac{448}{0.8} = \$560.00 \quad \text{(Original cost)}$$

Question 8 **[A]**



The investment has interest compounding annually and is credited annually.

Question 9 **[B]**

To calculate the monthly repayment

```
 N=60
 I%=9.2
 PV=18000
▪PMT=-375.40000…
 FV=0
 P/Y=12
 C/Y=12
 PMT:END BEGIN
```

After the tenth repayment

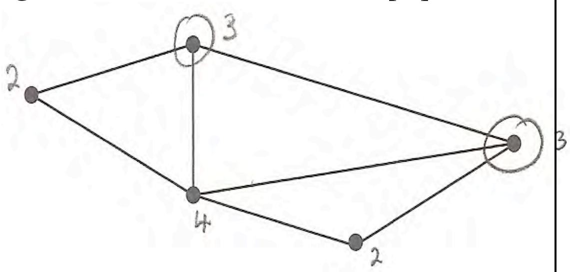
```
 N=10
 I%=9.2
 PV=18000
 PMT=-375.40000…
▪FV=15542.39977
 P/Y=12
 C/Y=12
 PMT:END BEGIN
```

$\$18000 - 15542.40 = \$2457.60$

Jenny has paid \$2457.60 of the principal immediately following the tenth repayment.

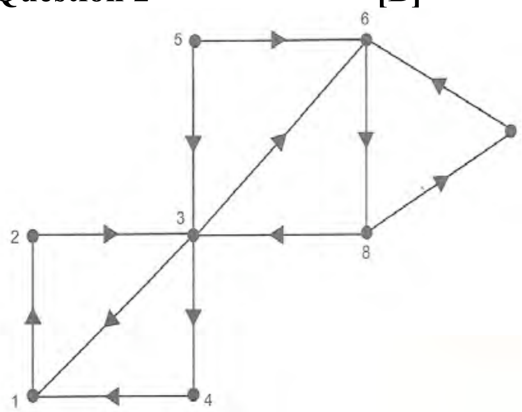
Module 5 Networks and decision mathematics

Question 1 [B]



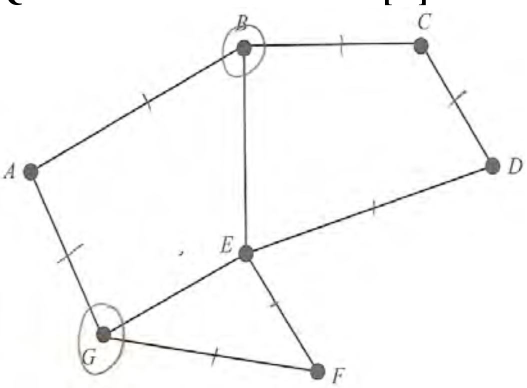
The number of vertices with an odd degree is 2.

Question 2 [B]



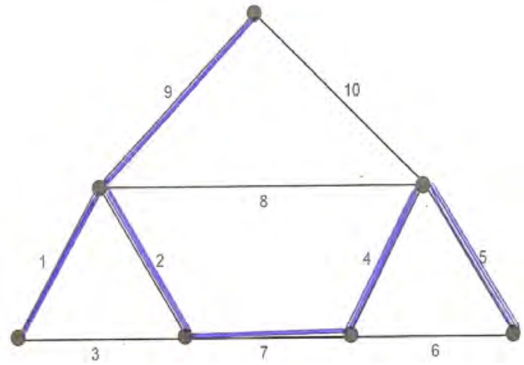
All intersections can be reached from intersection 5.

Question 3 [D]



As there are two vertices with odd degree, at least two Eulerian paths exist.

Question 4 [D]

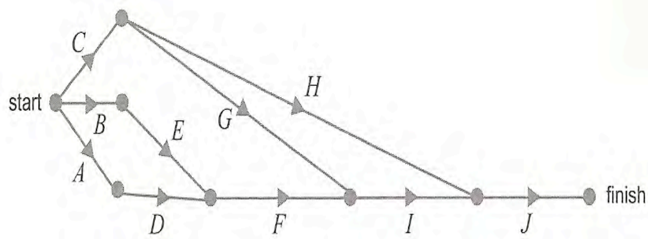


The minimal spanning tree for the network will include the edge that has the weight of 9.

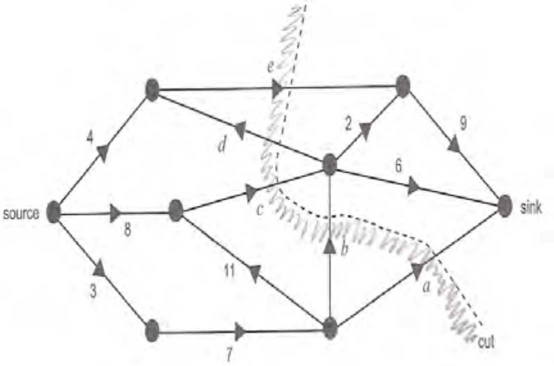
Question 5 [E]

Activity	Immediate predecessors
A	–
B	–
C	–
D	A
E	B
F	D, E
G	C
H	C
I	F, G
J	H, I

The directed graph which represents this project is:



Question 6 **[C]**



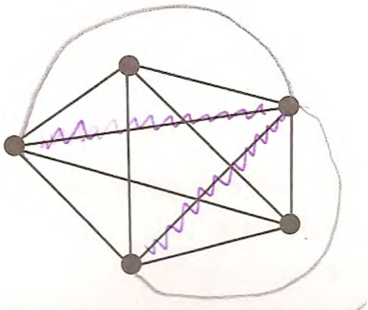
The capacity of the cut is $e + c + b + a$.
Note: d is heading back into the cut towards the source, so is not included.

Question 7 **[C]**

A six-team basketball competition where all teams play each other once would represent a complete graph with six vertices.

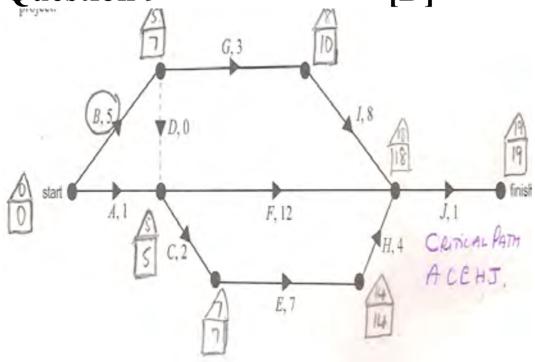
Question 8 **[D]**

Euler's formula applies to planar graphs which have no edges crossing.



This diagram cannot be drawn without edges crossing.

Question 9 **[D]**



If activity B is reduced by 4 hours, the project completion time is also reduced by 4 hours.

Module 6 Matrices

Question 1 **[B]**

The matrix $\begin{bmatrix} 12 & 36 \\ 0 & 24 \end{bmatrix} = 12\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$

Question 2 **[D]**

A=$\begin{bmatrix} -2 \\ 0 \end{bmatrix}$, B=$\begin{bmatrix} 0 & 9 \end{bmatrix}$, C=$\begin{bmatrix} 2 \end{bmatrix}$

The order of B is 1×2; the order of C is 1×1. If BC is defined then the number of columns of B must equal the number of rows of C.
Order BC = $(1 \times \mathbf{2}) \times (\mathbf{1} \times 1)$ ….this is not so and therefore the matrix BC is not defined.

Question 3 [A]

$A=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, B=\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix}, C=\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}$

Now A is the 2 × 2 unit matrix and so

A^3 is also $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$$B - C = \begin{bmatrix}2 & 1\\1 & 0\end{bmatrix} - \begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} = \begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}$$

$$A^3(B - C) = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}$$

Question 4 [B]

	Athletics	Cross country	Swimming
2004	Green	Green	Blue
2005	Green	Red	Blue
2006	Blue	Green	Blue

The total number of competitions won by each if the three teams in each of these three years is

$$\begin{array}{c} \\ 2004 \\ 2005 \\ 2006 \end{array}\begin{array}{c} \begin{array}{ccc} B & G & R \end{array} \\ \begin{bmatrix}1 & 2 & 0\\1 & 1 & 1\\2 & 1 & 0\end{bmatrix}\end{array}$$

Question 5 [D]

The new price matrix

$$MP = \begin{bmatrix}1.2 & 0\\0 & 1.35\end{bmatrix}\begin{bmatrix}145 & 210 & 350\\185 & 270 & 410\end{bmatrix}$$

$$= \begin{bmatrix}1.2\times145+0 & 1.2\times210+0 & 1.2\times350+0\\0+1.35\times185 & 0+1.35\times270 & 0+1.35\times410\end{bmatrix}$$

$$= \begin{bmatrix}174 & 252 & 420\\249.75 & 364.50 & 553.50\end{bmatrix}$$

Question 6 [E]

$$XA = X\begin{bmatrix}1 & 3\\6 & 4\\0 & 0\end{bmatrix} = \begin{bmatrix}4 & 1\\1 & 4\\3 & 5\end{bmatrix}$$

Let the order of X be $m \times n$. The order of A *is* 3 × 2.

Order XA $= (m \times n) \times (3 \times 2)$

$= (m \times 2)$

For this to exist then $n = 3$.

But $(m \times 2) = (3 \times 2)$ and so $m = 3$ as well.

Question 7 [C]

For simultaneous equations to have a unique solution, the determinant cannot be zero.

$4x+2y=10$	$x=0$	$x-y=3$	$2x+y=5$	$x=8$
$2x+y=5$	$x+y=6$	$x+y=3$	$2x+y=10$	$y=2$
(1)	(2)	(3)	(4)	(5)

1. $\det(1) = \begin{vmatrix}4 & 2\\2 & 1\end{vmatrix} = 4 - 4 = 0$

2. $\det(2) = \begin{vmatrix}1 & 0\\1 & 1\end{vmatrix} = 1 - 0 = 1$

3. $\det(3) = \begin{vmatrix}1 & -1\\1 & 1\end{vmatrix} = 1 - (-1) = 2$

4. $\det(4) = \begin{vmatrix}2 & 1\\2 & 1\end{vmatrix} = 2 - 2 = 0$

5. $\det(5) = \begin{vmatrix}1 & 0\\0 & 1\end{vmatrix} = 1 - 0 = 1$

There are 3 sets of simultaneous equations with a unique solution.

Question 8 [E]

95% of those who had their last holiday in Australia would have their next holiday in Australia.
20% of those who had their last holiday in Australia would have their next holiday overseas.
The transition matrix for this situation is $\begin{bmatrix} 0.95 & 0.80 \\ 0.05 & 0.20 \end{bmatrix}$

Question 9 [B]

The movement of birds is described by the transition matrix:

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0.8 & 0 \\ 0.2 & 1 \end{bmatrix} \end{array}$$

In the long term, the number of birds that settle at location A will gradually decrease to zero. This can be confirmed by multiplying this matrix by itself over and over again.